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$$(1) \quad \sin 2\beta = \sin AOC = \sin ACD \cdot \frac{AC}{R} = \frac{AC \cdot AD}{2R^2} = 2 \frac{AC}{2R} \cdot \frac{AD}{2R} = 2 \sin \beta \cos \beta.$$

$$(2) \quad \cos 2\beta = \cos AOC = \frac{R^2 + R^2 - \overline{AC}^2}{2R \cdot R} = \frac{4R^2 - \overline{AC}^2 - \overline{AC}^2}{4R^2} = \frac{\overline{AD}^2 - \overline{AC}^2}{4R^2}$$

$$= \left( \frac{AD}{2R} \right)^2 - \left( \frac{AC}{2R} \right)^2 = \cos^2 \beta - \sin^2 \beta.$$

$$(3) \quad \sin 3\beta \sin AEC = \sin AEO = \sin \beta \cdot \frac{R}{OE}.$$

Now

$$\frac{OE}{R} = \frac{AE}{AD} = \frac{AE}{2R \cos \beta}; \quad \text{i. e., } AE = 2\overline{OE} \cos \beta.$$

Again

$$OE^2 = \overline{AE}^2 + R^2 - 2\overline{AE} \cdot R \cos \beta = 4\overline{OE}^2 \cos^2 \beta + R^2 - 4\overline{OE} \cdot R \cdot \cos^2 \beta;$$

$$\therefore OE = \frac{R}{4 \cos^2 \beta - 1};$$

$$\therefore \sin 3\beta = \sin \beta (4 \cos^2 \beta - 1) = 3 \sin \beta - 4 \sin^3 \beta.$$

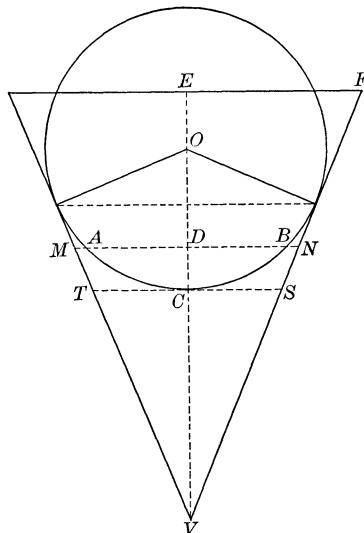
$$(4) \quad \cos 3\beta = \frac{R^2 - \overline{AE}^2 - \overline{OE}^2}{2\overline{OE} \cdot AE} = \frac{(4 \cos^2 \beta - 1)^2 - 1 - 4 \cos^2 \beta}{4 \cos \beta} = 4 \cos^3 \beta - 3 \cos \beta.$$

#### 462. Proposed by DANIEL KRETH, Wellman, Iowa.

A conical glass, the diameter of the base of which is 5 inches and altitude 6 inches, is one-fifth full of water. If a sphere 4 inches in diameter is dropped into it, how much of the vertical axis of the glass is immersed?

SOLUTION BY J. A. CAPRON, Notre Dame University.

Let  $MN$  be the level of the water after the sphere is dropped;  $R = 5/2$  inches = radius of base of given cone;  $r = 2$  inches = radius of sphere;  $r_1$  = radius of base of cone  $VMN$ ;  $x$  = height of spherical segment  $ACB$ ;  $\alpha$  = semivertical angle of cone; and  $CV = a$ .



Then volume of cone  $VMN = V_c = \frac{1}{3}\pi r_1^2(a+x)$ ; volume of segment  $ACB = V_s = \pi x^2(r-x/3)$ ; and volume of given cone  $= V = \frac{1}{3}\pi R^2 h$ , where  $h = 6$  inches.

Hence  $V_c - V_s = \frac{1}{5}V$ , since  $V_c - V_s$  = volume of water. Substituting, we get

$$r_1^2(a+x) - x^2(3r-x) = \frac{R^2h}{5}.$$

Since

$$VC = VO - CO \quad \text{and} \quad VO = \frac{r}{\sin \alpha},$$

we get

$$a = r \left( \frac{1}{\sin \alpha} - 1 \right),$$

but since

$$\tan \alpha = \frac{5}{12}, \quad \sin \alpha = \frac{5}{13},$$

therefore

$$a = 2 \left( \frac{13}{5} - 1 \right) = \frac{16}{5}.$$

From triangles  $VEF$  and  $VDN$ , we get

$$r_1 = \frac{(a+x)}{h} R, \quad \text{or} \quad r_1 = \frac{\left( \frac{16}{5} + x \right) \frac{5}{2}}{6} = \frac{4}{3} + \frac{5x}{12}.$$

Substituting these values in the above equation and reducing, we get

$$845x^3 - 3120x^2 + 3840x - 1304 = 0.$$

To solve this equation, let

$$x = y + \frac{108}{169}, \quad \text{or} \quad x = y + \frac{16}{13}.$$

Substituting and reducing, we get

$$169y^3 = -\frac{16^3}{13} + 260.8, \quad \text{or} \quad 13^3y^3 = -705.6.$$

Hence,

$$y = -\frac{\sqrt[3]{705.6}}{13} = -\frac{2}{13}\sqrt[3]{88.2}.$$

Then

$$x = y + \frac{16}{13} = \frac{16 - 2\sqrt[3]{88.2}}{13} = .54595.$$

Hence, the required height =  $a + x = 3.2 + .54595 = 3.74595$  inches.

Excellent solutions were received from NATHAN ALTSCHILLER, C. N. SCHMALL, HERBERT N. CARLETON, J. W. CLAWSON, HORACE OLSON, and PAUL CAPRON.

#### CALCULUS.

##### 375. Proposed by V. M. SPUNAR, Chicago, Illinois.

Solve the differential equation

$$x^2(a - bx) \frac{d^2y}{dx^2} - 2x(2a - bx) \frac{dy}{dx} + 2(3a - bx)y = 6a^2.$$

##### I. SOLUTION BY H. T. BIGELOW, La Fayette, Indiana.

Let

$$y = \sum_{n=0}^{\infty} c_n x^n, \quad \frac{dy}{dx} = \sum_{n=0}^{\infty} n c_n x^{n-1}, \quad \frac{d^2y}{dx^2} = \sum_{n=0}^{\infty} n(n-1) c_n x^{n-2}.$$

Substituting and collecting terms we have,

$$\sum_{n=0}^{\infty} a(n^2 - 5n + 6) c_n x^n \equiv \sum_{n=0}^{\infty} b(n^2 - 3n + 2) c_n x^{n+1} + 6a^2.$$